Chapter 4

Determination of plasma resistivity

The ability to reconstruct the axial dependence of the magnetic field in the vicinity of the probes due to plasma motion can now be put to use as tool to explore some fundamental physics.

How does the magnetic field evolve in space and time? How does that evolution constrain the effective electrical resistivity of the plasma? How does that measured value of resistivity compare with standard theoretical predictions. If the magnetic field and plasma velocity field are known functions of space and time then the effective resistivity of the plasma, in principle, can be deduced according to

$$\partial_t \mathbf{B} - \nabla\times(\mathbf{v}\times\mathbf{B}) = \eta\,\nabla^2\,\mathbf{B} \tag{4.1}$$

Where η is the electrical resistivity of the fluid. Alternative, we could more directly find the resistivity using a Ohm's law.

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta\mathbf{j} \tag{4.2}$$

It is clear that these are equivalent to each other since the curl of (4.2) equals (4.1). Then to find the resistivity we just need to evaluate the above combinations of the known quantities. But first we must take notice of the vector nature of the above equations.

In truth η is not really just a number, in the presence of a strong magnetic field its true

properties are best described as a tensor that takes into account the difference of plasma conductivity in the direction parallel to the magnetic field, as compared to its value in the direction perpendicular to the field. This can be phrased in terms the convenient laboratory-framed cylindrical coordinates (r, θ, z), but with some complications. Let η be a tensor-valued function of position

$$\eta = \begin{pmatrix} \eta_r(\vec{x}) & 0 & 0 \\ 0 & \eta_\theta(\vec{x}) & 0 \\ 0 & 0 & \eta_z(\vec{x}) \end{pmatrix} \tag{4.3}$$

where the dependence on position is simply an artifact due to choosing coordinates that are not locally parallel to $\mathbf{B}$ everywhere. The spatial dependence is purely determined by $\eta(\vec{x}) \cdot \mathbf{B}(\vec{x}) = \eta_{\|}\mathbf{B}$, where $\eta_{\|}$ is the parallel resistivity of the plasma, which should be a scalar constant that is approximately uniform over the volume of our system.

What is left over in the η tensor is due to the transverse resistivity $\eta_\perp$ of the plasma